1. The figure above shows the graph of $f''$, the derivative of $f$, on the closed interval $-1 \leq x \leq 5$. The graph of $f''$ has horizontal tangent lines at $x = 1$ and $x = 3$. The function $f$ is twice differentiable with $f(2)=6$.

(a) Find the $x$-coordinate of each of the points of inflection of the graph of $f$. Give a reason for your answer.
(b) At what value of $x$ does $f$ have a relative minimum value. Give a reason for your answer.
(c) State the interval(s) on which the graph of $f$ is decreasing. Give a reason for your answer.
(d) Let $g$ be the function defined by $g(x)=xf(x)$. Find an equation for the line tangent to the graph of $g$ at $x = 2$.

2. Consider the function $f(x) = x\sqrt{k-x}$, where $k$ is a real number.

a. Find $f'(x)$ and $f''(x)$.

b. For what constant $k$ does $f(x)$ have a relative maximum at $x = 2$.
3. Let \( f \) be a function with a second derivative given by \( f''(x) = x^2(x - 3)(x - 6) \). What are the \( x \)-coordinates of the points of inflection of the graph of \( f \)?

(A) 0 only  
(B) 3 only  
(C) 0 and 6 only  
(D) 3 and 6 only  
(E) 0, 3, and 6

<table>
<thead>
<tr>
<th>( x )</th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g'(x) )</td>
<td>2</td>
<td>3</td>
<td>0</td>
<td>-3</td>
<td>-2</td>
<td>-1</td>
<td>0</td>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>

4. The derivative \( g' \) of a function \( g \) is continuous and has exactly two zeros. Selected value of \( g' \) are given in the table above. If the domain of \( g \) is the set of all real numbers, then \( g \) is decreasing on which of the following intervals?

(A) \(-2 \leq x \leq 2\) only  
(B) \(-1 \leq x \leq 1\) only  
(C) \(x \geq -2\)  
(D) \(x \geq 2\) only  
(E) \(x \leq -2\) or \(x \geq 2\)

5. The radius of a sphere is decreasing at a rate of 2 centimeters per second. At the instant when the radius of the sphere is 3 centimeters, what is the rate of change, in square centimeters per second, of the surface area of the sphere? (The surface area \( S \) of a sphere with radius \( r \) is \( S = 4\pi r^2 \).)

(A) \(-108\pi\)  
(B) \(-72\pi\)  
(C) \(-48\pi\)  
(D) \(-24\pi\)  
(E) \(-16\pi\)

6. Let \( f \) be the function with derivative given by \( f'(x) = \sin(x^2 + 1) \). How many relative extrema does \( f \) have on the interval \( 2 < x < 4? \)

(A) One  
(B) Two  
(C) Three  
(D) Four  
(E) Five
7. Let $f$ be a twice-differentiable function such that $f(2) = 5$ and $f(5) = 2$. Let $g$ be the function given by $g(x) = f(f(x))$.

(a) Explain why there must be a value $c$ for $2 < c < 5$ such that $f'(c) = -1$.

(b) Show that $g'(2) = g'(5)$. Use this result to explain why there must be a value $k$ for $2 < k < 5$ such that $g''(k) = 0$.

(c) Show that if $f''(x) = 0$ for all $x$, then the graph of $g$ does not have a point of inflection.

(d) Let $h(x) = f(x) - x$. Explain why there must be a value $r$ for $2 < r < 5$ such that $h(r) = 0$. 